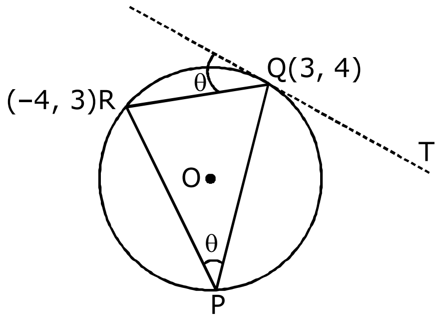
EXERCISE – V HINTS & SOLUTIONS

Sol.1 **(a)** $x^2 + y^2 = 25$

$m_{QR} = \frac{1}{7}$, $m_T = - \frac{-3}{4}$



$$\tan\theta = \left|\frac{\frac{1}{7} - \left(-\frac{3}{4}\right)}{1 + \left(\frac{1}{7}\right)\left(-\frac{3}{4}\right)}\right| = \left|\frac{4+21}{28-3}\right|$$

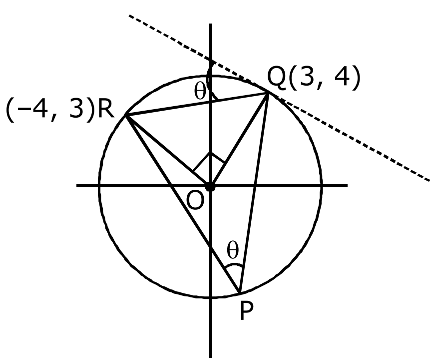
$= \frac{25}{25} = 1 \Rightarrow \tan\theta = 1$

$\theta = 45^o = \frac{\pi}{4}$

Aliter :

$m_{OQ} \cdot m_{OR} = -1$

$OQ \perp OR$



$\therefore$ angle substended by a chord at centre is double of the angle substanded at point on the opposite side of cirlce.

$\angle QPR = \frac{\pi}{4}$

(b) $x^2 + y^2 + 2x + 2ky + 6 = 0$

$x^2 + y^2 + 2ky + k = 0$

cuts orthogonally

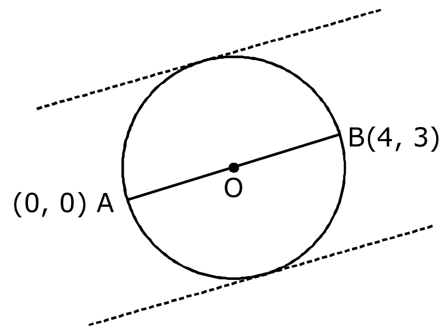
$\Rightarrow$ $2.1.0 + 2.k.k = 6 + k$

$\Rightarrow$ $2k^2 - k - 6 = 0$

$(k - 2)(2k + 3) = 0 \ k = 2, -\frac{3}{2}$

Sol.2 **(a)** Centre $\left(2, \frac{3}{2}\right)$, $r = \frac{5}{2}$

$m_{AB} = \frac{3}{4}$



$\therefore$ Tangent $3x - 4y + \lambda = 0$

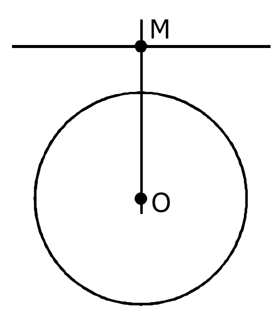
$\frac{5}{2} = \frac{|6 - 6 + \lambda|}{\sqrt{25}} \Rightarrow \frac{5}{2} = |\lambda| \Rightarrow \lambda = \pm \frac{25}{2}$

tangents are $6x - 8y \pm 25 = 0$

(b) $16(x^2 + y^2) + 32x - 8y - 50 = 0$

$x^2 + y^2 + 2x - \frac{1}{2}y - \frac{25}{8} = 0$

centre $\left(-1, \frac{1}{4}\right)$



& line $y = 2x + 11 \Rightarrow 2x - y + 11 = 0$

$\perp$ line is $x + 2y + \lambda = 0$

passing through centre $\left(-1, \frac{1}{4}\right)$

$-1 + \frac{1}{2} + \lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

$\perp$ line is $2x + 4y + 1 = 0$

Intersection point M is $\left(\frac{-9}{2}, 2\right)$

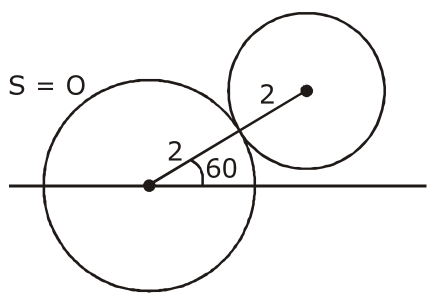
(c) $S \equiv x^2 + y^2 + 4x = 0$

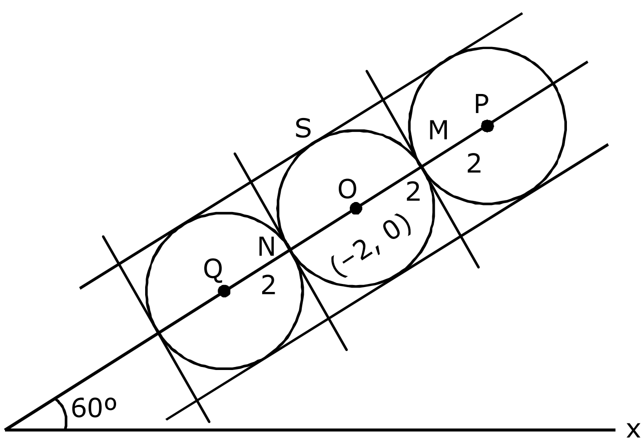
centre $(-2, 0)$, $r = 2$

circle is

$(x + 2)^2 + y^2 = 4^2$

$x^2 + y^2 + 4x - 12 = 0$





Line passing $(-2, 0)$ & $\theta = 60^\circ$

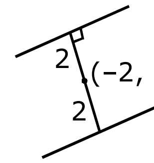
$$\frac{x+2}{\frac{1}{2}} = \frac{y}{\frac{\sqrt{3}}{2}} = \pm 4$$

$x = \pm 2 - 2$, $y = 2\sqrt{3}$

$P(0, 2\sqrt{3})$ & $Q(-4, -2\sqrt{3})$

D.C.T. tangent parallel to line joining centres.

($\because$ radius is equal to both circles)



$\sqrt{3}x - y + \lambda = 0$

$$2 = \frac{|-2\sqrt{3} + \lambda|}{2} \Rightarrow \lambda = 2\sqrt{3} \pm 4$$

$\sqrt{3}x - y + 2\sqrt{3} \pm 4 = 0$

T.C.T. $\perp$ lines passes M & N

M & N are $(-1, \sqrt{3})$ & $(-3, -\sqrt{3})$

$x + \sqrt{3}y + \mu = 0$

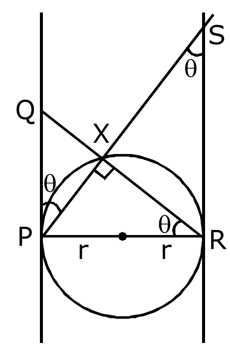
$-1 + 3 + \mu = 0 \qquad \Rightarrow \mu = -2$

$\Rightarrow \quad x + \sqrt{3}y - 2 = 0$

& $-3 - 3 + \mu = 0 \Rightarrow \mu = 6 \Rightarrow x + \sqrt{3}y + 6 = 0$

Sol.3 **(a)** In ΔPQR

$$\frac{PQ}{2r} = \tan\theta$$



& In ΔPRS

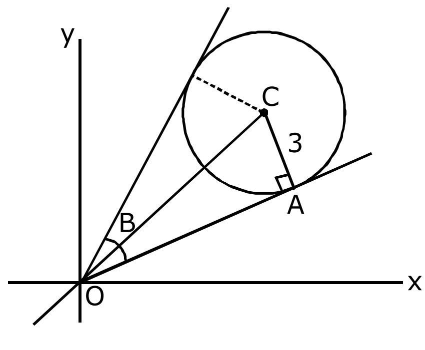
$$\frac{PR}{RS} = \tan\theta$$

$\Rightarrow \quad (2r)^2 = (PQ) . (RS)$

$\Rightarrow \quad 2r = \sqrt{(PQ).(RS)}$

(b) Pair of lines

$2x^2 - 3xy + y^2 = 0$



$$\tan\theta = \frac{2\sqrt{\frac{9}{4}-2}}{3} = \frac{1}{3}$$

$$\sin\theta = \frac{1}{\sqrt{10}}, \cos\theta = \frac{3}{\sqrt{10}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = \sqrt{10} - 3$$

In ΔOAC

$$\tan\frac{\theta}{2} = \frac{3}{OA}$$

$$\Rightarrow OA = \frac{3}{\tan\left(\frac{\theta}{2}\right)} = \frac{3}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$$

$$= 3(\sqrt{10} + 3)$$

Sol.4 **(a)** $(x^2 + y^2 - 2x - 6y + 6)$
$+ \lambda (x^2 + y^2 + 2x - 6y + 6) = 0$
$(1 + \lambda)(x^2 + y^2) + 2x(\lambda - 1)$
$- 6y(\lambda + 1) + 6(\lambda + 1) = 0$

$$\Rightarrow \quad x^2 + y^2 + 2x\frac{(\lambda-1)}{(\lambda+1)} - 6y + 6 = 0$$

Cuts orthogonally $x^2 + y^2 + 4x + 6y + 4 = 0$

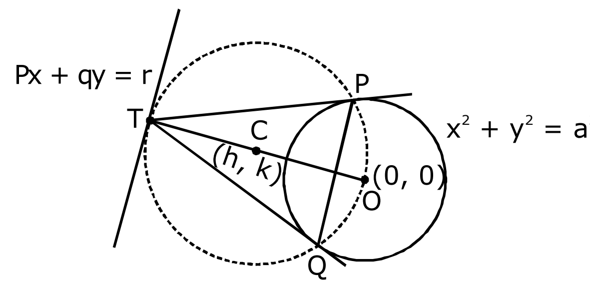
$$2.\frac{(\lambda-1)}{(\lambda+1)}.2 + 2.(-3)(3) = 6 + 4$$

$$\Rightarrow \quad \frac{2(\lambda-1)}{(\lambda+1)} - 9 = 5 \Rightarrow \frac{\lambda-1}{\lambda+1} = 7$$

$x^2 + y^2 + 14x - 6y + 6 = 0$

(b) C is mid point of OT
$2(CT) = OT$

$$\frac{2|Ph + qk - r|}{\sqrt{p^2+q^2}} = \frac{|-r|}{\sqrt{p^2+q^2}}$$



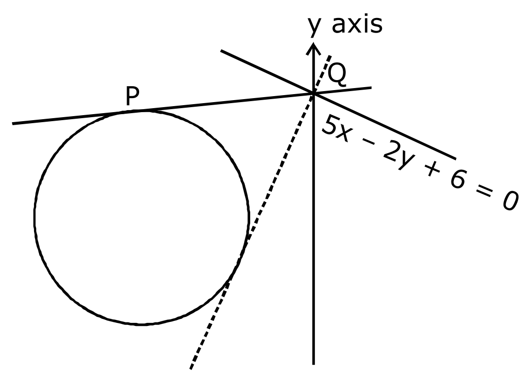
C & O lie same side of line

$\therefore$ $2(Ph + qk - r) = -r$
$2px + 2qy = r$

Sol.5 **(a)** $x^2 + y^2 + 6x + 6y - 2 = 0$
$Q \equiv (0, 3)$

$PQ = \sqrt{\delta_1} = \sqrt{0+9+18-2} = 5$

(b) $x > 2b > 0$
$x^2 + y^2 = b^2, (x - a)^2 + y^2 = b^2$



common tangent $y = mx - b\sqrt{1+m^2}$

$$b = \frac{|am - b\sqrt{1+m^2}|}{\sqrt{m^2+1}}$$

$\Rightarrow$ $b^2(m^2 + 1) = (am - b\sqrt{1+m^2})^2$

$\Rightarrow$ $b^2(m^2 + 1) = a^2m^2 + b^2(1 + m^2)$
$- 2abm\sqrt{1+m^2}$

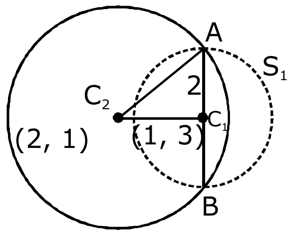
$m \neq 0$ or $(ma)^2 = (2b) = (\sqrt{1+m^2})^2$

$\because$ $m > 0$ $m^2(a^2 - 4b^2) = 4b^2$

$$\Rightarrow \quad m = \frac{2b}{\sqrt{a^2-4b^2}}$$

Sol.6 $S_1 \equiv x^2 + y^2 - 2x - 6y + 6 = 0$

centre r_1 (1, 3), $r_2 = 2$



$r_2 = \sqrt{2^2 + (C_1C_2)^2}$

$(C_1C_2)^2 = 1^2 + 2^2 = 5$

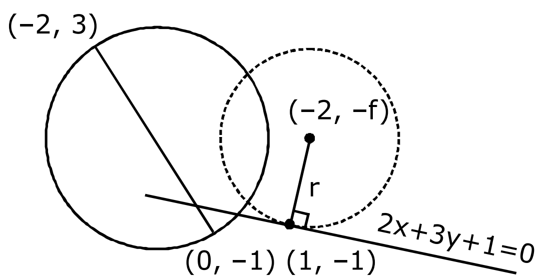
$r_2 = \sqrt{9} = 3$

Sol.7 $x(x + 2) + (y + 1)(y - 3) = 0$

$x^2 + y^2 + 2x - 2y - 3 = 0$

& $x^2 + y^2 + 2gx + 2fy + c = 0$

are orthogonal



$2g + 2f(-1) = c - 3$

$2g - 2f + 3 = c$ ....(i)

circle passing through

(1, −1) $(2g - 2f) + c + 2 = 0$

$\Rightarrow \quad c - 3 + c + 2 = 0$

$\Rightarrow \quad c = \frac{1}{2}$

tangent at (1, −1)

$x - y + 2(x + 1) + f(y - 1) + c = 0$

$\Rightarrow x(g + 1) + y(f - 1) + g - f + c = 0$

$\Rightarrow x(g + 1) + y(f - 1) + \frac{c}{2} + \frac{3}{2} + c = 0$

$\Rightarrow x(g + 1) + y(f - 1) + - \frac{3}{4} + c = 0$

& $2x + 3y + 1 = 0$ are same tangent

$$\frac{g+1}{2} = \frac{f-1}{3} = \frac{-3/4}{1}$$

$\Rightarrow g = \frac{-5}{2}$ & $f = -\frac{-5}{4}$

circle is

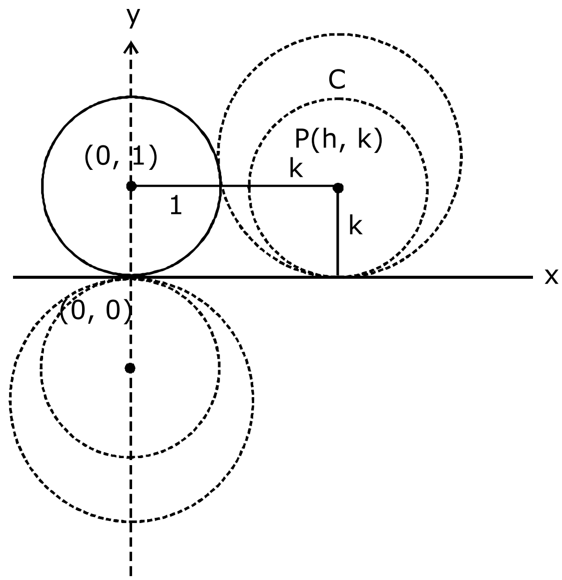
$$x^2 + y^2 - 5x - \frac{5}{2}y + \frac{1}{2} = 0$$

$\Rightarrow 2x^2 + 2y^2 - 10x - 5y + 1 = 0$

Sol.8 $r_1 = 1, r_2 = k$

centre

(0, 1) & (h, k)



$k + 1 = \sqrt{h^2 + (k-1)^2}$

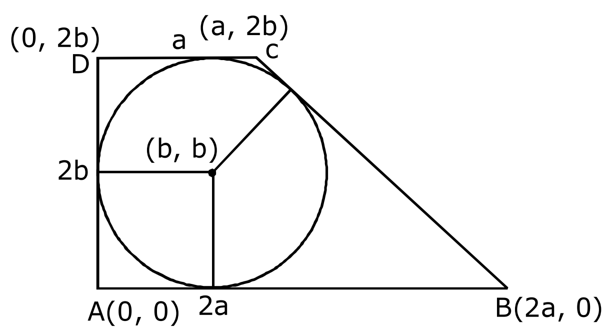
$k^2 + 2k + 1 = h^2 + k^2 - 2k + 1$

$\Rightarrow \quad h^2 - 4k = 0 \quad$ locus $x^2 = 4y$

Aliter :

$18 = \frac{1}{2}(2a + a) \times 2b$

$6 = ab$



Line BC is tangent to $(x - b)^2 + (y - b)^2 = b^2$

$y = \frac{2b}{-a}(x - 2a) \Rightarrow 2bx + ay - 4ab = 0$

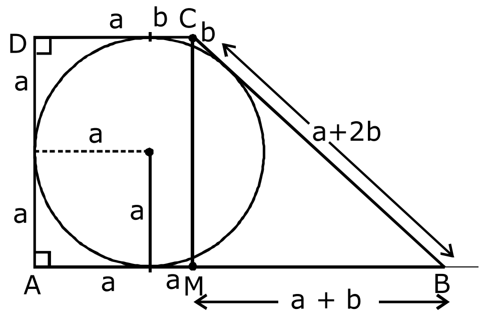
$$b = \frac{|2b^2 + ab - 4ab|}{\sqrt{4b^2 + a^2}}$$

$b^2 (4b^2 + a^2) = (2b^2 - 18)^2$
$4b^4 + a^2b^2 = 4b^4 - 72b^2 + 324$
$72b^2 = 324 - 324$
$72b^2 = 288$
$b^2 = 4 \Rightarrow b = 2$
All circle touches the given circle which centre lie on (–y) axis
centres coordinates $(0, y)\ \forall\ y \le 0$

Sol.9 **(a)** AB = 2CD

$$18 = \frac{1}{2}[AB + CD] \times AD$$

$$18 = \frac{1}{2}[3(a + b)] \times 2a$$



$\Rightarrow$ $a(a + b) = 6$ ....(i)

or $a^2 + ab = 6$ or $b = \frac{6 - a^2}{a}$

In ΔBCM
$(2a)^2 + (a + b)^2 = (a + 3b)^2$
$4a^2 + a^2 + b^2 + 2ab = a^2 + 9b^2 + 6ab$
$\Rightarrow 4a^2 = 4ab + 8b^2 \Rightarrow a^2 = ab + 2b^2$
from (i)
$a^2 = 6 - a^2 + 2b^2 \Rightarrow b^2 = a^2 - 3$ ...(ii)
$\Rightarrow 9a^2 = 36 \Rightarrow a^2 = 4 \Rightarrow a = 2$

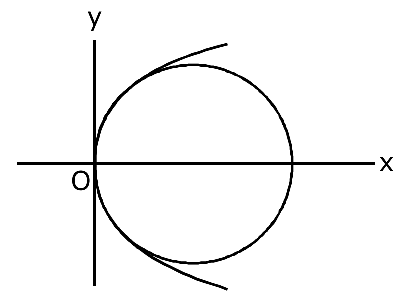
(b) $x^2 + y^2 = 169$, (17, 7)
S-I If (0, 0) & (17, 7) have distance is equal to $\sqrt{2}/3$ distance $= \sqrt{17^2 + 49}$

$= \sqrt{289 + 49}$

$= \sqrt{338} = \sqrt{2 \times 169} = 13\sqrt{2}$

Pt (17, 7) lie on director circle.
S-I is true
S-II $x^2 + y^2 = (\sqrt{2}/3)^2$
$\Rightarrow x^2 + y^2 = 338$ S-II is true

Sol.10 (a) $C_1 : y^2 = 4x$ & $C_2 : x^2 + y^2 - 6x + 1 = 0$
Intersection point



$x^2 + 4x - 6x + 1 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0 \Rightarrow x = 1$
$y = \pm 2$ (1, 2) & (1, –2)
Touch exactly two points
(b) $L_1 : 2x + 3y + p - 3 = 0$
$L_2 : 2x + 3y + p + 3 = 0$
$C : x^2 + y^2 + 6x - 10y + 30 = 0$
centre (–3, 5), r = 2
distance between lines

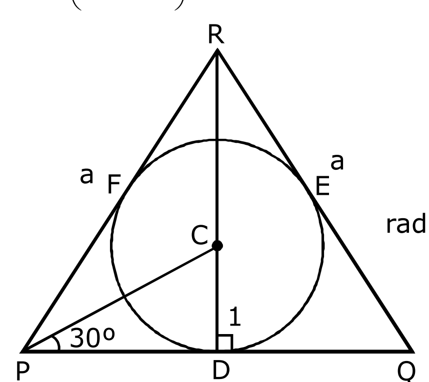
$$d = \frac{|p + 3 - p + 3|}{\sqrt{13}} = \frac{6}{\sqrt{13}} < 2$$

S-I true & S-II is false

(c) Comprehension

$L_{PQ} \equiv \sqrt{3}x + y - 6 = 0$

$D\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$



radius = 1 = r

$L_{PQ}(0) < 0$ & $L_{PQ}(c) < 0$

$r = \dfrac{\Delta}{s} \Rightarrow \Delta = s \Rightarrow \dfrac{\sqrt{3}a^2}{4} = \dfrac{3a}{2} \Rightarrow a = 2\sqrt{3}$

(i) Line CD should be a coordinate c

$$\frac{x - \frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{y - \frac{3}{2}}{\frac{1}{2}} = \pm 1$$

$x = \dfrac{3\sqrt{3}}{2} \pm \dfrac{\sqrt{3}}{2}$ $\quad y = \dfrac{3}{2} \pm \dfrac{1}{2}$ C is

$(2\sqrt{3}, 2)$ or $(\sqrt{3}, 1)$

$L > 0$ $\quad L < 0$

$C(\sqrt{3}, 1)$

Circle is

$(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) $\dfrac{x - 3\sqrt{3}/2}{\sqrt{3}/2} = \dfrac{y - 3/2}{1/2} = -3$ RD = 3

$x = 0, y = 0$ $\quad R(0, 0)$

Line PQ

$$\frac{x - \frac{3\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{y - \frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \pm\sqrt{3}$$ P & Q are

$Q(\sqrt{3}, 3)$ & $P(2\sqrt{3}, 0)$

E & F are mid point

$E\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)$ & $F(\sqrt{3}, 0)$

(iii) Lines : RP & PQ are

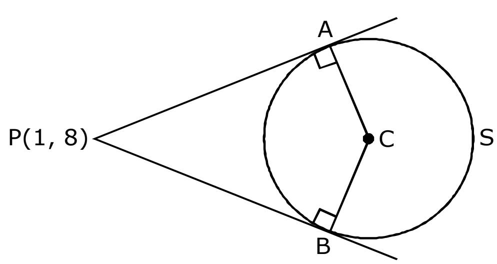
$y = \dfrac{3x}{\sqrt{3}}$ and $y = \dfrac{0x}{2\sqrt{3}}$

$y = \sqrt{3}x$ & $y = 0$

Sol.11 (a) $x^2 + y^2 - 6x - 4y - 11 = 0$

PACD is a cyclic quadrialteral

C(3, 2)



circle as diamter PC

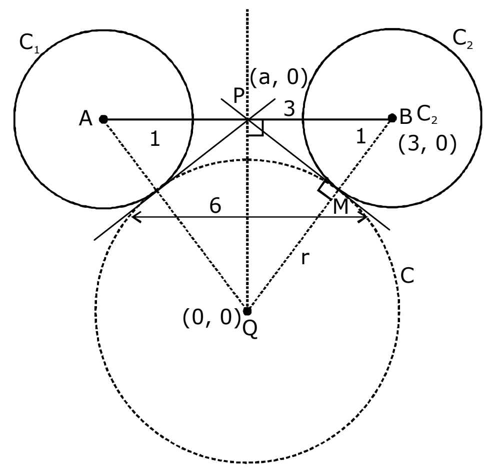
$(x - 1)(x - 3) + (y - 8)(y - 2) = 0$

$x^2 - 4x + 3 + y^2 - 10y + 16 = 0$

$\Rightarrow$ $x^2 + y^2 - 4x - 10y + 19 = 0$

(b) $(PM)^2 = 3^2 - 1^2 = 8$

$PQ^2 = (r + 1)^2 - 3^2$



In ΔPMQ

$PQ^2 = (PM)^2 + r^2$

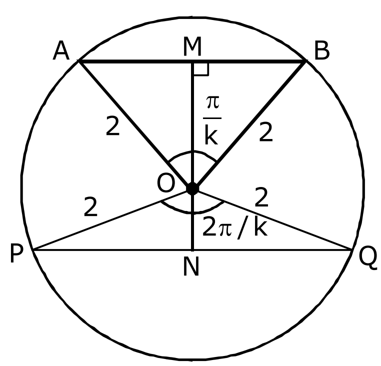
$= 8 + r^2$

$r^2 + 2r + 1 - 9 = 8 + r^2$

$2r = 16$

$r = 8$

Sol.12 AB||PQ, MN = $\sqrt{3} + 1$



$OM = 2\cos\dfrac{\pi}{2k}$ & $ON = 2\cos\dfrac{\pi}{k}$

$OM + ON = \sqrt{3} + 1$

$2\left(\cos\dfrac{\pi}{2k} + \cos\dfrac{\pi}{k}\right) = \sqrt{3} + 1$

$$2\left(\cos\frac{\theta}{2}+\cos\theta\right) = \sqrt{3} + 1 \quad \text{Let } \frac{\pi}{k} = \theta$$

$$2\cos\frac{\theta}{2} + 4\cos^2\frac{\theta}{2} - 2 = \sqrt{3} + 1$$

$$4\cos^2\frac{\theta}{2} + 2\cos\frac{\theta}{2} - (\sqrt{3} + 3) = 0$$

$$\cos\frac{\theta}{2} = -\frac{-2\pm\sqrt{4+16(\sqrt{3}+3)}}{4}$$

$$= \frac{-1\pm\sqrt{1+2.1.2\sqrt{3}+(2\sqrt{3})^2}}{2}$$

$$= \frac{-1\pm(2\sqrt{3}+1)^2}{2} = -\frac{-2(\sqrt{3}+1)}{2} \text{ or } \frac{\sqrt{3}}{2}$$

$$= -(\sqrt{3}+1) \text{ or } \frac{\sqrt{3}}{2}$$

$$\cos\frac{\theta}{2} \neq -(\sqrt{3}+1) > 1$$

$$\therefore \quad \text{COS}\frac{\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3} = \frac{\pi}{k}$$

$$\Rightarrow \quad k = 3$$

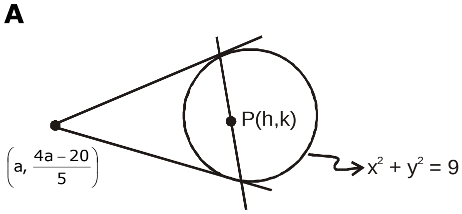
Sol.13 (D)

Equation of circle using family of circle $(x - a)^2 + (y - 2)^2 + \lambda x = 0$
put $(-1, 0)$ to get λ & check from options

Sol.14 0002

Origin lies left to the line. Points (2, 3/4) & (1/4, –1/4) lie in the smaller part & also in the circle so only two points.

Sol.15 A



Equation of chord of contact

$$ax + \left(\frac{4a-20}{5}\right)y - 9 = 0$$

$5ax + 4ay - 20y - 45 = 0$
$5ax + (4a - 20)\,y - 45 = 0$ .....(i)

equation of chord of mid point
$hx + ky = h^2 + k^2$ .....(ii)

$$\frac{5a}{h} = \frac{4a-20}{k} = \frac{45}{h^2+k^2} \Rightarrow a = \frac{9h}{h^2+k^2}$$

$$4a - 20 = \frac{45k}{h^2+k^2} \text{ put the value of a}$$

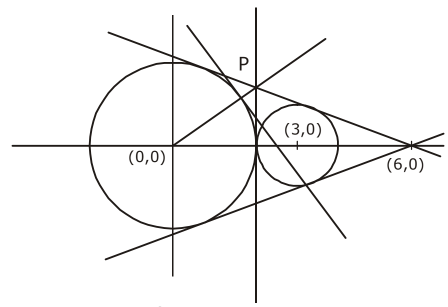
$$\frac{36h}{h^2+k^2} \quad 20 = \frac{45k}{h^2+k^2}$$

$36x - 20(x^2 + y^2) = 45y$
$20(x^2 + y^2) + 45y - 36x = 0$

Sol.16 A

$y = (1/\sqrt{3})\,(x - 3) \pm \sqrt{1+1/3}$

Sol.17 D



(6, 0) satisfies so only option D is correct